**CN #5 Algebraic Proof**

**Vocabulary**

A **proof** is an argument that uses logic, definitions, properties, and previously proven statements to show that a conclusion is true.

An important part of writing a proof is giving justifications to show that every step is valid.

**Properties of Equality**

<table>
<thead>
<tr>
<th>Property</th>
<th>Statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition Property of Equality</td>
<td>If $a = b$, then $a + c = b + c$.</td>
</tr>
<tr>
<td>Subtraction Property of Equality</td>
<td>If $a = b$, then $a - c = b - c$.</td>
</tr>
<tr>
<td>Multiplication Property of Equality</td>
<td>If $a = b$, then $ac = bc$.</td>
</tr>
</tbody>
</table>
| Division Property of Equality  | If $a = b$ and $c 
eq 0$, then $\frac{a}{c} = \frac{b}{c}$. |
| Reflexive Property of Equality | $a = a$                                        |
| Symmetric Property of Equality | If $a = b$, then $b = a$.                      |
| Transitive Property of Equality | If $a = b$ and $b = c$, then $a = c$.          |
| Substitution Property of Equality| If $a = b$, then $b$ can be substituted for $a$ in any expression. |

**Remember!**

The Distributive Property states that

$$a(b + c) = ab + ac.$$
Example 1: Solving an Equation in Algebra

Solve the equation $4m - 8 = -12$. Write a justification for each step.

\[
\begin{align*}
4m - 8 &= -12 & \text{Given equation} \\
\underline{+8} & \quad +8 & \text{Addition Property of Equality} \\
4m &= -4 & \text{Simplify using Addition} \\
\frac{4m}{4} &= \frac{-4}{4} & \text{Division Property of Equality} \\
m &= -1 & \text{Simplify using Division}
\end{align*}
\]

Abbreviations: Given; Add. Prop. =, Div. Prop. =

Check It Out! Example 1

Solve the equation $\frac{1}{2}t = -7$. Write a justification for each step.

\[
\begin{align*}
\frac{1}{2}t &= -7 & \text{Given equation} \\
2 \left( \frac{1}{2} \right) t &= 2(-7) & \text{Multiplication Property of Equality.} \\
t &= -14 & \text{Simplify using mult.}
\end{align*}
\]

Check It Out! Example 2

What is the temperature in degrees Celsius $C$ when it is $86^\circ F$? Solve the equation $C = \frac{5}{9}(F - 32)$ for $C$ and justify each step.

Check It Out! Example 2 Continued

Understand the Problem

The answer will be the temperature in degrees Celsius.

List the important information:

\[
C = \frac{5}{9}(F - 32) \quad F = 86
\]
Make a Plan
Substitute the given information into the formula and solve.

Check It Out! Example 2 Continued

Solve

\[ C = \frac{5}{9}(F - 32) \quad \text{Given equation} \]

\[ C = \frac{5}{9}(86 - 32) \quad \text{Substitution Property of Equality} \]

\[ C = \frac{5}{9}(54) \quad \text{Simplify using subtraction.} \]

\[ C = 30 \quad \text{Simplify using multiplication.} \]

\[ C = 30^\circ \quad \text{Use proper notation/units} \]

Look Back
Check your answer by substituting it back into the original formula.

\[ C = \frac{5}{9}(F - 32) \]

\[ 30 = \frac{5}{9}(86 - 32) \]

\[ 30 = 30 \checkmark \]

Check It Out! Example 2 Continued

Like algebra, geometry also uses numbers, variables, and operations. For example, segment lengths and angle measures are numbers. So you can use these same properties of equality to write algebraic proofs in geometry.

Helpful Hint

\[ \overline{AB} \]

\[ A \quad \quad B \]

\( \overline{AB} \) represents the length \( \overline{AB} \), so you can think of \( \overline{AB} \) as a variable representing a number.
Example 3: Solving an Equation in Geometry

Solve for x. Write a justification for each step.

\[ NO = NM + MO \]  \hspace{1cm} \text{Segment Addition Postulate}

\[ 4x - 4 = 2x + (3x - 9) \]  \hspace{1cm} \text{Substitution Property of Equality}

\[ 4x - 4 = 5x - 9 \]  \hspace{1cm} \text{Simplify using Addition}

\[ -4 = x - 9 \]  \hspace{1cm} \text{Subtraction Property of Equality}

\[ 5 = x \]  \hspace{1cm} \text{Addition Property of Equality}

Check It Out! Example 3

Solve for x. Write a justification for each step.

\[ m\angle ABC = m\angle ABD + m\angle DBC \]  \hspace{1cm} \text{Angle Add. Post.}

\[ 8x^\circ = (3x + 5)^\circ + (6x - 16)^\circ \]  \hspace{1cm} \text{Subst. Prop. of Equality}

\[ 8x = 9x - 11 \]  \hspace{1cm} \text{Simplify}

\[ -x = -11 \]  \hspace{1cm} \text{Subtr. Prop. of Equality}

\[ x = 11 \]  \hspace{1cm} \text{Mult. Prop. of Equality}

You learned in Chapter 1 that segments with equal lengths are congruent and that angles with equal measures are congruent. So the Reflexive, Symmetric, and Transitive Properties of Equality have corresponding properties of congruence.

### Properties of Congruence

<table>
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<tr>
<th>SYMBOLS</th>
<th>EXAMPLE</th>
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<tbody>
<tr>
<td>Reflexive Property of Congruence</td>
<td>( EF \equiv EF )</td>
</tr>
<tr>
<td>Symmetric Property of Congruence</td>
<td>If figure A ( \equiv ) figure B, then figure B ( \equiv ) figure A, (Sym. Prop. of ( \equiv ))</td>
</tr>
<tr>
<td>Transitive Property of Congruence</td>
<td>If figure A ( \equiv ) figure B and figure B ( \equiv ) figure C, then figure A ( \equiv ) figure C, (Trans. Prop. of ( \equiv ))</td>
</tr>
</tbody>
</table>

You learned in Chapter 1 that segments with equal lengths are congruent and that angles with equal measures are congruent. So the Reflexive, Symmetric, and Transitive Properties of Equality have corresponding properties of congruence.
Remember!
Numbers are equal (=) and figures are congruent (≡).

Example 4: Identifying Property of Equality and Congruence

Identify the property that justifies each statement.

A. \( \triangle QRS \equiv \triangle QRS \)  Reflex. Prop. of Congruency
B. \( m\angle 1 = m\angle 2 \) so \( m\angle 2 = m\angle 1 \)  Symm. Prop. of =
C. \( \overline{AB} \equiv \overline{CD} \) and \( \overline{CD} \equiv \overline{EF} \) so \( \overline{AB} \equiv \overline{EF} \)  Trans. Prop of =
D. \( 32^\circ = 32^\circ \)  Reflex. Prop. of =

Check It Out! Example 4

Identify the property that justifies each statement.

4a. \( DE = GH \), so \( GH = DE \).  Symm. Prop. of =
4b. \( 94^\circ = 94^\circ \)  Reflex. Prop. of =
4c. \( 0 = a \), and \( a = x \). So \( 0 = x \).  Trans. Prop. of =
4d. \( \angle A \equiv \angle Y \), so \( \angle Y \equiv \angle A \)  Symm. Prop. of =

Lesson Quiz: Part I

Solve each equation. Write a justification for each step.

1. \( \frac{z-5}{6} = -2 \)
   \[
   \frac{z-5}{6} = -2 \quad \text{Given}
   \]
   \[
   z - 5 = -12 \quad \text{Mult. Prop. of =}
   \]
   \[
   z = -7 \quad \text{Add. Prop. of =}
   \]
Lesson Quiz: Part II
Solve each equation. Write a justification for each step.

2. \(6r - 3 = -2(r + 1)\)

\[
\begin{align*}
6r - 3 &= -2(r + 1) & \text{Given} \\
6r - 3 &= -2r - 2 & \text{Distrib. Prop.} \\
8r - 3 &= -2 & \text{Add. Prop. of } = \\
8r &= 1 & \text{Add. Prop. of } = \\
r &= \frac{1}{8} & \text{Div. Prop. of } =
\end{align*}
\]

Lesson Quiz: Part III
Identify the property that justifies each statement.

3. \(x = y\) and \(y = z\), so \(x = z\). \(\text{Trans. Prop. of } =\)

4. \(\angle DEF \cong \angle DEF\). \(\text{Reflex. Prop. of } \cong\)

5. \(\overline{AB} \cong \overline{CD}, \text{ so } \overline{CD} \cong \overline{AB}\). \(\text{Sym. Prop. of } \cong\)