CN#2 Conditional Statements

OBJECTIVES:

I WILL BE ABLE TO IDENTIFY, WRITE, AND ANALYZE THE TRUTH VALUE OF CONDITIONAL STATEMENTS.

I WILL BE ABLE TO WRITE THE INVERSE, CONVERSE, AND CONTRAPOSITIVE OF A CONDITIONAL STATEMENT.

Venn Diagrams

Recall that in a Venn diagram, ovals are used to represent each set. The ovals can overlap if the sets share common elements. The real number system contains an infinite number of subsets. The following chart shows some of them.

<table>
<thead>
<tr>
<th>Set</th>
<th>Description</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural numbers</td>
<td>The counting numbers</td>
<td>1, 2, 3, 4, 5, ...</td>
</tr>
<tr>
<td>Whole numbers</td>
<td>The set of natural numbers and 0</td>
<td>0, 1, 2, 3, 4, ...</td>
</tr>
<tr>
<td>Integers</td>
<td>The set of whole numbers and their opposites</td>
<td>..., -2, -1, 0, 1, 2, ...</td>
</tr>
<tr>
<td>Rational numbers</td>
<td>The set of numbers that can be written as a ratio of integers</td>
<td>-3/4, 5, -2, 0.5, 0</td>
</tr>
<tr>
<td>Irrational numbers</td>
<td>The set of numbers that cannot be written as a ratio of integers</td>
<td>π, √10, 6 + √2</td>
</tr>
</tbody>
</table>

Vocabulary

conditional statement
hypothesis
conclusion
truth value
negation
converse
inverse
contrapositive
logically equivalent statements

By phrasing a conjecture as an if-then statement, you can quickly identify its hypothesis and conclusion.
Check It Out! Example 1

Identify the hypothesis and conclusion of the statement.
"A number is divisible by 3 if it is divisible by 6."
Hypothesis: A number is divisible by 6.
Conclusion: A number is divisible by 3.

Check It Out! Example 2

Write a conditional statement from the sentence “Two angles that are complementary are acute.”

Two angles that are complementary are acute.

If two angles are complementary, then they are acute.

Writing Math

“If $p$, then $q$” can also be written as “if $p$, $q$,” “$q$, if $p$,” “$p$ implies $q$,” and “$p$ only if $q$.”

Many sentences without the words if and then can be written as conditionals. To do so, identify the sentence’s hypothesis and conclusion by figuring out which part of the statement depends on the other.

A conditional statement has a truth value of either true (T) or false (F). It is false only when the hypothesis is true and the conclusion is false.

To show that a conditional statement is false, you need to find only one counterexample where the hypothesis is true and the conclusion is false.
### Example 3A: Analyzing the Truth Value of a Conditional Statement

Determine if the conditional is true. If false, give a counterexample.

If this month is August, then next month is September.

When the hypothesis is true, the conclusion is also true because September follows August. So the conditional is true.

### Example 3B: Analyzing the Truth Value of a Conditional Statement

Determine if the conditional is true. If false, give a counterexample.

If two angles are acute, then they are congruent.

You can have acute angles with measures of 80° and 30°. In this case, the hypothesis is true, but the conclusion is false.

Since you can find a counterexample, the conditional is false.

### Example 3C: Analyzing the Truth Value of a Conditional Statement

Determine if the conditional is true. If false, give a counterexample.

If an even number greater than 2 is prime, then $5 + 4 = 8$.

An even number greater than 2 will never be prime, so the hypothesis is false.

$5 + 4$ is not equal to 8, so the conclusion is false.

However, the conditional is true because the hypothesis is false.

### Check It Out! Example 3

Determine if the conditional “If a number is odd, then it is divisible by 3” is true. If false, give a counterexample.

An example of an odd number is 7. It is not divisible by 3. In this case, the hypothesis is true, but the conclusion is false. Since you can find a counterexample, the conditional is false.
The negation of statement $p$ is “not $p$,” written as $\neg p$. The negation of a true statement is false, and the negation of a false statement is true.

Remember!
If the hypothesis is false, the conditional statement is true, regardless of the truth value of the conclusion.

The negation of statement $p$ is “not $p$,” written as $\neg p$. The negation of a true statement is false, and the negation of a false statement is true.

### Related Conditionals

<table>
<thead>
<tr>
<th>Definition</th>
<th>Symbols</th>
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<tbody>
<tr>
<td>A conditional is a statement that can be written in the form “If $p$, then $q$.”</td>
<td>$p \rightarrow q$</td>
</tr>
<tr>
<td>The converse is the statement formed by exchanging the hypothesis and conclusion.</td>
<td>$q \rightarrow p$</td>
</tr>
<tr>
<td>The inverse is the statement formed by negating the hypothesis and conclusion.</td>
<td>$\neg p \rightarrow \neg q$</td>
</tr>
<tr>
<td>The contrapositive is the statement formed by both exchanging and negating the hypothesis and conclusion.</td>
<td>$\neg q \rightarrow \neg p$</td>
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### Conditional Statement
A type of logical statement that consists of two parts: hypothesis and conclusion.

Example: (Can use the “If-then” form)
If class is fun, then it must be math!!

hypothesis conclusion

### Converse
A statement formed by switching the hypothesis and conclusion of a conditional statement.

Example:
If class is fun, then it is math!!

Converse (switch):
If it is math, then class is fun!!

hypothesis conclusion

hypothesis conclusion
Negation

The negative or opposite of a statement, can be written 2 ways.
Example #1:
- Statement: I like ice cream.
- Negation: I don’t like ice cream.
Example #2:
- Statement: I don’t like ice cream.
- Negation: I do like ice cream.

Inverse

A logical statement where the hypothesis and conclusion are both negated.
Example:
- If class is fun, then it is math.
- Inverse (negate):
  If class is not fun, then it is not math.

Contrapositive

A statement formed by both negating and switching the hypothesis and conclusion of a conditional statement.
Example:
- If class is fun, then it is math.
  Contrapositive (switch and negate):
  If it is not math, then class is not fun.

Logically Equivalent Statements

Related conditional statements that have the same truth value are called logically equivalent statements.
Two statements that are either both true or both false.
- Conditional = Contrapositive
- Inverse = Converse

Helpful Hint
The logical equivalence of a conditional and its contrapositive is known as the Law of Contrapositive.
Write the converse, inverse, and contrapositive of the conditional statement “If an animal is a cat, then it has four paws.”* Find the truth value of each.

What is the conditional statement?

If an animal is a cat, then it has four paws.*

*For logic’s sake, we will not discuss ALL possible scenarios in the animal kingdom. We realize there are cats in existence that have lost a paw.

Check It Out! Example 4
If an animal is a cat, then it has four paws.

Converse (switch):
If an animal has 4 paws, then it is a cat.
There are other animals that have 4 paws that are not cats, so the converse is false.

Inverse (negate):
If an animal is not a cat, then it does not have 4 paws.
There are animals that are not cats that have 4 paws, so the inverse is false.

Contrapositive (switch & negate):
If an animal does not have 4 paws, then it is not a cat.
Cats have 4 paws, so the contrapositive is true.