## Construct Regular Polygons

In Chapter 4, you learned that an equilateral triangle is a triangle with three congruent sides. You also learned that an equilateral triangle is equiangular, meaning that all its angles are congruent.

In this lab, you will construct polygons that are both equilateral and equiangular by inscribing them in circles.

## Activity 1

(1) Construct circle $P$. Draw a diameter $\overline{A C}$.
(2) Construct the perpendicular bisector of $\overline{A C}$. Label the intersections of the bisector and the circle as $B$ and $D$.
(3) Draw $\overline{A B}, \overline{B C}, \overline{C D}$, and $\overline{D A}$. The polygon $A B C D$ is a regular quadrilateral. This means it is a four-sided polygon that has four congruent sides and four congruent angles.


## Try This

1. Describe a different method for constructing a regular quadrilateral.
2. The regular quadrilateral in Activity 1 is inscribed in the circle. What is the relationship between the circle and the regular quadrilateral?
3. A regular octagon is an eight-sided polygon that has eight congruent sides and eight congruent angles. Use angle bisectors to construct a regular octagon from a regular quadrilateral.

## Activity 2

(1) Construct circle $P$. Draw a point $A$ on the circle.
(2) Use the same compass setting. Starting at $A$, draw arcs to mark off equal parts along the circle. Label the other points where the arcs intersect the circle as $B, C, D, E$, and $F$.
(3) Draw $\overline{A B}, \overline{B C}, \overline{C D}, \overline{D E}, \overline{E F}$, and $\overline{F A}$. The polygon $A B C D E F$ is a regular hexagon. This means it is a six-sided polygon that has six congruent
 sides and six congruent angles.

## Try This

4. Justify the conclusion that $A B C D E F$ is a regular hexagon. (Hint: Draw diameters $\overline{A D}, \overline{B E}$, and $\overline{C F}$. What types of triangles are formed?)
5. A regular dodecagon is a 12 -sided polygon that has 12 congruent sides and 12 congruent angles. Use the construction of a regular hexagon to construct a regular dodecagon. Explain your method.

## Activity 3

(1) Construct circle $P$. Draw a diameter $\overline{A B}$.
(2) Construct the perpendicular bisector of $\overline{A B}$. Label one point where the bisector intersects the circle as point $E$.
(3) Construct the midpoint of radius $\overline{P B}$. Label it as point $C$.
(4) Set your compass to the length $C E$. Place the compass point at $C$ and draw an arc that intersects $\overline{A B}$. Label the point of intersection $D$.
(5) Set the compass to the length $E D$. Starting at $E$, draw arcs to mark off equal parts along the circle. Label the other points where the arcs intersect the circle as $F, G$, $H$, and $J$.
(6) Draw $\overline{E F}, \overline{F G}, \overline{G H}, \overline{H J}$, and $\overline{J E}$. The polygon $E F G H J$ is a regular pentagon. This means it is a five-sided polygon that has five congruent sides and five congruent angles.


Steps 1-3


Step 4


Step 5


Step 6

## Iry This

6. A regular decagon is a ten-sided polygon that has ten congruent sides and ten congruent angles. Use the construction of a regular pentagon to construct a regular decagon. Explain your method.
7. Measure each angle of the regular polygons in Activities 1-3 and complete the following table.

## REGULAR POLYGONS

| Number of Sides | 3 | 4 | 5 | 6 |
| :--- | :---: | :---: | :---: | :---: |
| Measure of Each Angle | $60^{\circ}$ |  |  |  |
| Sum of Angle Measures | $180^{\circ}$ |  |  |  |

8. Make a Conjecture What is a general rule for finding the sum of the angle measures in a regular polygon with $n$ sides?
9. Make a Conjecture What is a general rule for finding the measure of each angle in a regular polygon with $n$ sides?
